The Geometry of Variational Principles on Grassmann Fibrations

Z. Urban

The Grassmann fibrations are regarded in this lecture as basic geometric underlying structures for variational problems for submanifolds. The corresponding variational objects (Lagrangian, Euler-Lagrange form, Helmholtz form, Noether current) appear in this context to be rather classes than differential forms as such. The notion of a Lepage form on Grassmann fibration is introduced. We consider the variational functionals associated with Lepage forms for higher-order immersed curves and first-order immersed submanifolds and discuss the inverse variational problem in terms of Helmholtz conditions. The correspondence with homogeneous variational problems on slit tangent bundles will be discussed.

References

[1]   M. Crampin, D. J. Saunders, The Hilbert-Carathéodory Form for Parametric Multiple Integral Problems in the Calculus of Variations, Acta Appl. Math. 76 (2003) 37−55.

[2]   P. Dedecker, On the generalization of symplectic geometry to multiple integrals in the calculus of variations, in: Lecture notes in Math. 570, Springer, Berlin, 1977, 395−456.

[3]   D. R. Grigore, D. Krupka, Invariants of velocities and higher-order Grassmann bundles, J. Geom. Phys. 24 (1998) 244–264.

[4]   D. Krupka, Lepagean forms in higher order variational theory, in: Modern Developments in Analytical Mechanics, Proc. IUTAM-ISIMM Sympos., Turin, June 1982, Academy of Sciences of Turin, 1983, 197-238.

[5]   D. Krupka, Lepage forms in Kawaguchi spaces and the Hilbert form, Publ. Math. Debrecen (2013), to appear.

[6]   Z. Urban, Variational sequences in mechanics on Grassmann fibrations, Ph.D. Dissertation, University of Ostrava, 2011, 75pp.

Zbyněk Urban

Lepage Research Institute, Slatinky, Czech Republic
and
Department of Mathematics and Physics, Faculty of Electrotechnics
and Informatics, University of Pardubice, Studentska 95, 532 10
Pardubice, Czech Republic

E-mail: zbynek.urban@lepageri.eu